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Theoretical Aspects of Minimax Automated Control of Uncertainty in Aviation Safety System

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Abstract
The study and analysis of risks in the sense of threats, dangers and negative consequences is impossible without a preliminary study of the uncertainties in which these risks are generated and formed, as well as realized or not realized. As a result, in many areas of knowledge there has been a need to consider uncertainty as an important concept in management theory. Just as today there is no single definition (as well as interpretation) of the concept of "risk" in the world, there is no single concept of "uncertainty". Intuitively, we call the inability to obtain objective, absolutely true, complete knowledge about the properties, functioning of the system, and the true values of its parameters, uncertainty. Similarly, today there is no clear understanding of the essence of the goals and objectives of uncertainty management. The purpose of the article is to study and research the theoretical aspects of managing the uncertainty of the evolution of the aviation safety system.

Keywords: uncertainty, automation, control, uncertainty management, aviation safety management system

INTRODUCTION

In this article, the aviation safety system is regarded as a specific dynamic system, which, as is known, is described by either differential or finite-difference recurrent relations. From a game-theoretical point of view, the system experiences purposeful control of its evolution to ensure the necessary and sufficient level of its functioning, while being under the independent influence of uncertain factors, which are usually called "nature" in game theory. The task of managing such a system, in particular, involves constructing a sequence of conscious actions with the system with confidence in the incompleteness, inaccuracy and unreliability of knowledge about evolution. Optimality of such control means achieving the guaranteed best control result with the most unfavorable "natural" "responses" of the system in turn.

In the classical theory of optimal control, it is believed that the initial phase state of the controlled dynamic system is known and true.

Then the control problem consists of finding a time function that minimizes (or maximizes) the quality functional of the control process. The solution to such a problem is based on the method of dynamic programming of R. Bellman or on the maximum principle of L. Pontryagin.

A more complex task is one in which the initial phase state is not accurate and involves the accumulation of "knowledge" about the controlled system in the process of its change ("movement"). In this case, random processes of errors in "knowledge" are either investigated (studied) or minimax game-theoretic approaches are used.

1. LITERATURE REVIEW AND PROBLEM STATEMENT

Significant results in the theory and methodology of studying uncertainty and its typification are pre-sented in works [1-10]. In [11], issues of strategic management of uncertainty and decision-making under conditions of uncertainty based on multi-criteria parameters are considered.

The interpretation of the concept of "uncertainty management" as minimizing uncertainty is presented in works [12-13].

From the point of view of systems analysis, the most common uncertainties are the uncertainty of goals, the uncertainty of knowledge about possible situations (situational uncertainty) and the uncer-tainty of conflicts.

The authors of this article agree with the opinion that uncertainty management cannot be identified with information management [14], which in turn cannot be identified with true knowledge. Therefore, the entropy toolkit for determining the value of information is not necessarily the only suitable one for managing the uncertainty of the system state.

Uncertainty management [15-16] can be interpreted as follows. Let the evolution of the aviation safety system be considered in a phase space in which information about the phase vector is insufficient, incomplete, questionable, or completely absent.

2. THE AIM AND OBJECTIVES OF THE STUDY

The purpose of the article is to study and research the theoretical aspects of managing the uncertainty of the evolution of the aviation safety system.

The uncertainty of changes in the state of the aviation safety system is the basis for studying the study and risk management [17] of aviation safety, as the next category of self-development of the managed system.

Uncertainty management involves, in particular, a transition from almost zero predictability of the behaviour of a security system, even in a short-term sense, to predictability in probabilistic and (or) possibilistic terms, depending on the chosen underlying theories (probability or possibilities).

Uncertainty management is carried out by continuous or discrete acquisition of reliable information (training and replenishment of knowledge) about the control object in order to obtain the necessary probabilistic or possibilistic characteristics of certain parameters of the evolution models of the controlled system.

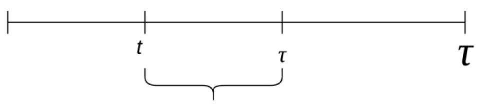
To achieve this goal, the following tasks were formulated:

1. Based on the game-theoretic approach to managing a dynamic system, select a theoretically substantiated form of setting the problem of managing an aviation safety system under conditions of uncertainty;
2. Present a mathematical formalization of the minimax model for managing the uncertainty of an aviation safety system with one control command and a possible principle of its use.
3. Show the possibility of generalizing the model for a multi-team dynamic system for managing the uncertainty of the evolution of the aviation safety system.

3. RESULTS OF INVESTIGATING THEORETICAL ASPECTS OF MINIMAX AUTOMATED CONTROL OF UNCERTAINTY IN AVIATION SAFETY SYSTEM

Let the aviation safety system evolve on its own. This system is controlled by one command c (), which can begin to operate at an arbitrary time t from a fixed evolution interval [0; T]. Let τ be time, к be the coordinate system.

The evolution of the system can then be described in the following way:



$$\left[\begin{array}{l} \begin{cases} k(\tau) = k_0 + \vartheta_0 \cdot \tau, \\ \quad \vartheta(\tau) = \vartheta_0 \end{cases} 0 \le \tau \le t \\ \begin{cases} k(\tau) = k_0 + \vartheta_0 \cdot \tau + (\tau - t) \cdot c, \\ \quad \vartheta(\tau) = \vartheta_0 + c, t \le \tau \le \mathrm{T} \end{cases} \end{array}\right.$$

The parameters t and k control the system subject to restrictions $0 \le \tau \le t$ *u* $|c| \le C$, where C>0- is the maximum absolute value of the control command. Let the evolution of the system be carried out according to the differential equation

$$k'' - \mathrm{c} \cdot \delta \cdot (\tau - t) = 0 \,,$$

where $\delta(\tau)$ is the Dirac delta function,

That is, in particular

$\int_{-\infty}^{+\infty} \delta(\tau) \cdot c(\tau) \cdot d\tau = c(0)$, and

$$\int_{-\infty}^{+\infty} \delta(\tau - t) \cdot c(\tau) \cdot d\tau = c(t)$$

The condition for the uncertainty of the initial values of the coordinates and the rate of evolution of the system with known approximations ¯K and ¯V V and given permissible errors $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ can be written as follows:

$$/k_0 - {}^{-}k^0 / \leq \varepsilon_{-1},$$

$$/\vartheta_0 - {}^{-}V^0 / \leq \varepsilon_{-2}$$

The condition for the uncertainty of the initial values of the coordinates and the rate of evolution of the system with known approximations and V and given permissible errors $k(\tau)$ and $V(\tau)$ can be written as follows:

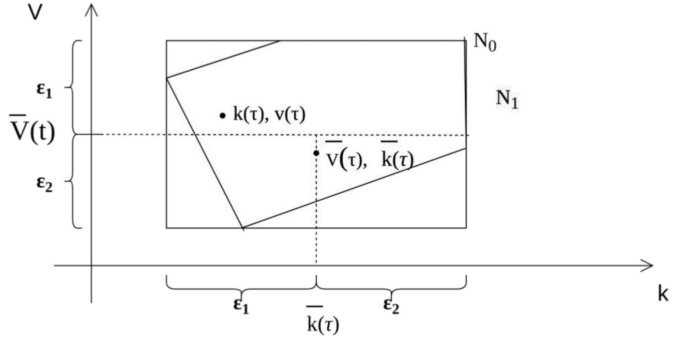
$$|k0 - k0| \leq \varepsilon 1,$$

$$|\vartheta 0 - V0| \leq \varepsilon 2$$

The evolution of the aviation safety system is continuously monitored, as a result of which the approximate values of the coordinates and evolution rates $k(\tau)$ and $V(\tau)$ become known, that is, the inequalities remain true:

$$\{|k(\tau) - k(\tau)| \leq \varepsilon 1, 0 \leq \tau m \leq \mathrm{T}(1)$$
$$|\vartheta(\tau) - V(\tau)| \leq \varepsilon 2$$

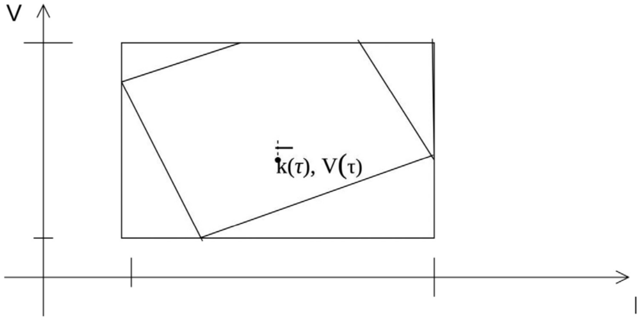
$$k(0) = k^0$$
$$V(0) = V0$$



Condition (1) is geometrically interpreted by the region N1 - convex subset of the rectangle N0. Continuous monitoring of the aviation safety system entails

$$\bigcap_{0\leq\theta\leq\tau} N\theta$$

The goal of managing the aviation safety system is the minimum guaranteed value of the functional $F=|k(\tau)|$ the meaning of which is to deviate from a given point $N\_(\ \tau)$, thus there are many possible states of the system at time τ.



For each moment of time, knowledge about the set of possible initial states $N_{\tau-1}$ is replenished and the moment of issuing the command $t(N,\tau)$ is calculated, and $\tau\leq t\leq T$ $\forall\tau\in[0;T]$ the value of command $c=c(N,\tau)$. When the equality $t(N,\tau)=\tau$ is reached, the command c(N,τ)is issued. Then it is natural to call the pair $\{t(N,\tau),c(N,\tau)\}$- a system control strategy, and the pair $\{t_{opt}(N,\tau),c_{opt}(N,\tau)\}_{\text{minmax}}$ – minmax an optimal minimax guaranteed control strategy.

If after τ - - the moment of time before the receipt of the control command using the strategy used, that is

$$t_{opt}(N,\tau)-\tau\geq0,\text{then } F_{min}|k(T)|=L(N,\tau)=min_t max_{k,V} min_c max_{k0,V0}|k(T)|,$$

$$k(T)=k_0+\vartheta_0\cdot\tau+(T-t)\cdot c$$

For $min_t, t\in[\tau,T]$,for $min_c,|c|\leq C$..

$\underset{k,V}{\underline{max}}$ is preserved according to the initial values of coordinates and speed,

that is,known values up to moment $k(d)$ and $\tau(d)$ for $d\in(\tau,t]$

$$N_t = N_\tau \cap N_{\tau,t}, \text{where } N_{\tau,t} = \{(k_0, \vartheta_0) | |k_0 + \vartheta_0(d) - u(d)| \le \varepsilon_1 | \vartheta_0 - V(d) | \le \varepsilon_2 | \ .$$

In this case, N_t is the condition for consistency of monitoring the state of the system.

The presented model (the simplest case) with a one-time command to control the uncertainty of the evolution of the aviation safety system is quite understandable and naturally generalized to the case of a multi-command controlled process, which can be specified by a recurrent relation (by system):

$$k_{n+1} = P(k_n, c_n, \tau_n) \ n \in 0,1 \ldots M$$
$$\tau_0 < \tau_1 < \ldots \tau_n < \tau_{n+1} \ \tau$$

K_n and P – h- dimensional expressions,

K_n, $P \in E^h$, where E^h -h- is an h-dimensional Euclidean space

Command cк -l- dimensional vector, $c_n P \in E^l$

On the other hand, $c_n \in C_n \subset E^l$ where C_n – are limited closed frequencies.

There is certain functionality in the trajectories of the system

$$F = f_{M+1}(C_{M+1}) + \Sigma f_i(c_i) \ _{Mi=0}$$

Minimum guaranteed value of the functionality

$$F_j = f_{M+1}(C_{M+1}) + \Sigma f_i(c_i) \ j \in [o; M] \cap N$$

can be found using the dynamic programming method by defining the Bellman function

$$L(N_j \tau_j) = \min_{c_j} \max_{B_{j+1}} \min_{c_{j+1}} \max_{B_{j+2}} \ldots \min_{c_M} \max_{B_{M+1}} \max_{k_{M+1}} F$$

where

$B_j \subset E_m$; B_j sets of possible outcomes of observations of the aviation safety system

4. DISCUSSION OF RESEARCH RESULTS OF THEORETICAL ASPECTS OF MINIMAX AUTOMATED CONTROL OF UNCERTAINTY IN AVIATION SAFETY SYSTEM

The authors of the article, in principle, support the current opinion that managing uncertainty consists of reducing its level, which in turn helps, in a certain sense, to reduce the level of certain types of risks inherent in the aviation safety system. The above becomes possible thanks to the emerging opportunity (owing to the acquisition of knowledge about the controlled system) to influence processes, events and even phenomena whose nature is not (or not fully) studied, understandable and explainable, and which manifest themselves under many types of uncertainty.

The continuously increasing number of scientific publications on uncertainty management indicates the relevance and relevance of the theoretical and methodological apparatus for managing uncertainty in systems that play an important role in the scientific, technical and technological development of production, transport, the service sector, etc. The authors of this article are not aware of similar publications on aviation safety. Considering aviation safety to be an integral part of the national security of Ukraine, the authors agree with the opinion that its provision today is mostly achieved through regulatory and organizational means.

The next possible step towards its strengthening could be the construction and continuous improvement of mathematical models, allowing, in particular, to optimize, modernize and continue to theoretically study the continuously evolving aviation safety system.

Mathematical modelling of this system will enrich the research and foster the improvement of technical, regulatory, resource and economic means of ensuring and continuously enhancing aviation safety.

To provide mathematical modelling for the purpose of a systematic approach to aviation safety management, a modern theoretical framework and its main (for a specific mathematical model) provisions and statements are naturally necessary.

When choosing a game-theoretic framework, the authors in this work examined, in particular, the issues of correctness of using the classical minimax principle. Minimizing the maximum deviation in a general sense serves as a well-known principle for the optimal choice of parameters of a mathematical model under construction, which allows the use of successive approximation methods to determine stationary points.

While not the only principle for solving these problems, this principle allows us to find a solution that has the alternance property for some classes of optimization problems [18-19].

The assessment of the state of an aviation safety system naturally depends on the internal characteristics of the system and on the external influence of "nature" (the environment), which can have both a positive and negative integral indicator. The properties of instability, heterogeneity, unevenness, and the patterns of external influence on the system, in particular, are both the cause and, to a certain extent, the explanation of the uncertainty of its state.

This article was written on the basis of the hypothesis that there is some "error" in the information about the phase vector of the system under study, as well as about accompanying the evolution of the system with continuous observation, analysis and understanding of its state. This, in turn, involves the use of modern automatic control systems and decision support systems, as well as highly organized management of the organization of automated management of aviation safety uncertainty.

SUMMARY

In this article, the following research problems were solved:

1. Based on the game-theoretic approach of a dynamic system, a theoretically substantiated form of setting the problem of managing an aviation safety system under conditions of uncertainty was chosen, the basis of which was insufficient, incomplete, or dubious information about the phase vector.
2. A mathematical formalization of the minimax model for managing the uncertainty of the evolution of an aviation safety system with one control command was presented.
3. The possibility of generalizing the model for a multi-command dynamic uncertainty control system using the dynamic programming method, in particular the Bellman function, was presented.

From the authors' point of view, the proposed approaches can make a significant contribution to the development of aviation safety management system tools in order to maintain global, national and regional acceptable levels of air industry safety.

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