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The Use of the Queueing Theory for the Analysis of Transport Processes

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This article focuses on the application of the queueing theory for the analysis of transport processes with regard to rail traffic. Selected aspects of queueing theory application for the analysis and rail traffic flow assessment are presented. A literature review regarding process modelling in rail transport using mass service theory models was performed. In order to study rail traffic flow, the application Java Modelling Tools- JSIM graph was used. The train movement process was analysed with regard to the traffic flow. The study was conducted on the basis of selected stretches of railway lines, numbers 2 (central long-distance in Warsaw) and 448 (central suburban in Warsaw).

Keywords: modelling processes, rail transport, mass service theory, traffic flow.

1. INTRODUCTION

Methods of solving decision problems in transport systems can be separated into optimization and simulation. In order to solve optimization tasks formulated using transport process models, it is necessary to apply appropriate method of searching in the solution domain. Conducting research in terms of various goals requires a behavioural representation of a system as well as its individual elements. In order to test dynamic transport models, it is necessary to build systems which simulate their operation. These efforts are undertaken since carrying out studies on real facilities is not possible, or there would be high costs associated with performing this type of research. One of the basic methods applied to the study of transport processes is the mass service theory, or in other words, the queueing theory. The queueing theory studies processes where the need to carry out certain tasks (services) arises on one side, and the need to fulfil them arises on the other side. This theory, encompassing so-called queue systems, has the objective to search for analytic dependencies. In turn, these dependencies allow us to set parameters characteristic of service processes completed in systems. Determination of mass service theory characteristics requires knowledge about the kind and type of service requests and types of service schedules.

Due to this fact, the mass system models may only be applied in few instances, however, for the remaining instances it is necessary to apply the simulation research technique.

For the classic mass service theory (MST) areas of application, we can include:

– rail traffic engineering, including planning train traffic on a single-track railway line, planning passenger train servicing at holding

stations or planning the train composition at marshalling yards in particular [9, 10, 11, 25, 27],
- determining the number of vehicles necessary to service a given region in variable demand conditions for transport [16, 17],
- optimizing the run frequency of vehicles based on the type of given route, taking into account the cost and completion quality of service [15, 16],
- setting the length of light signal cycle composition [2, 14],
- assessing the consequences of infrastructure parameter changes (for example: the number of traffic lanes or changes in geometric road parameters) on the delays arising during rush hour, as well as delays and congestion caused by traffic accidents and incidents involving elements of linear transport infrastructure [3, 13], etc.

Many researchers applied the mass service theory to describe railway transport processes. J. Leszczyński in publication [10] defined the term "process phase networks" (denoted in the form of simple mass service system networks) for the first time and introduced the possibility of applying this approach to plan railway traffic on a single-track railway line, planning passenger train servicing at holding stations or planning train composition at marshalling yards.

In the railway segment model presented in publication [11], elements of nodal infrastructure (railway stations) were modelled using TMO. A random railway station represented a system of mass service containing three subsystems: the station entrance, service at the station and the station exit.

In articles [28, 30] J. Żak indicated that there exists a possibility of applying the process phase network method (containing TMO) in order to solve problems arising not only in railway traffic engineering, but also problems from other transport processes. J. Woch also used the mass service theory to model processes in railway transport. In publication [25], he suggested applying mobile buffer models and the maximum expected traffic flow.

In research papers on this topic, authors most often assumed that requests in transport processes have the character of Poisson's schedule, however Węgierski [22] and Woch used various schedules in railway traffic engineering. In this paper, models containing process phase networks were used for analysis [1, 6, 11, 29].

The subject of this article is application of the queueing theory to assess the completion of transport processes with regard to rail traffic. The process of train movement based on traffic flow was analysed. The study was conducted on selected stretches of railway lines, numbers 2 (central long-distance in Warsaw) and 448 (central suburban in Warsaw).

2. APPLICATION OF MASS SERVICE THEORY TO STUDY RAILWAY TRANSPORT PROCESSES

Creating queues in railway transport results from factors depending on:

- the infrastructure: implementing interim speed limits (not included in the timetable- exceeded traffic product at the train crossing , damage to the track, track bed and other structures), introducing intensive modernization work and emergency track repairs, traffic control device malfunction, a break in the conductor rail, among others,
- the superstructures: vehicle with different parameters than in the timetable intended for servicing a train, no possibility to make use of the full power of the vehicle, vehicle malfunction, driving the vehicle against the style imposed in the timetable (prolonged driving time),
- the carrier: prolonged exchange of travellers, unplanned exchange of conductor or contact line crews, prolonged customs clearance at the border, realization of communication conditions, delayed notification of readiness to depart, foreign train delay,
- other reasons: associated with traffic engineering (late permission signal projection on the semaphore, incorrect traffic organization, lack of crew at the post, etc.), associated with the timetable (incorrect construction, transport of oversize or hazardous cargo), accidents (train, with vehicles, with people), elements (atmospheric events, fires, floods and others) and other events (emergency service intervention, strikes, terrorist threats), thefts, devastations and others.

The factors listed above are the reason why it is essential to use stochastic parameters and characteristics- variable in time, on the base of which steering algorithms are constructed and which are used to model infrastructure and transport processes- to define queue problems in railway transport systems. One of the methods which considers the stochastic character of events is the theory of mass service, or in other words, queueing theory.

The essence of the mass service theory is made up of the following elements:

1. Request – the queueing theory assumes a random nature of service request creation, i.e. the time interval between adjacent moments of incoming requests into the system is a positive random variable. Other studied quantities are similar in nature.
2. Input request stream – the sequence of consecutive time intervals between adjacent moments of requests received by the system.
3. Service device – a person or device which performs services. The service time is also a positive random variable, and similarly to the input request stream, it is assumed that service devices work independently of each other and the service time of each device has the same timetable.
4. Queue – the queue in the service system arises when it is not possible to service all requests at a given time since all service devices are occupied at that time.

The basic condition for applying analytical methods is the assumption that the request stream is:

- a stationary process – the probability of incoming requests depends only on the length of time intervals, but does not depend on their position on the timeline,
- memoryless – the probability of k requests in a given time interval does not depend on the number of requests and how they occurred up to that point,
- single – two or more requests cannot appear at the same time,

Formulating the queue model, the following should be defined:

- type of probability distribution of random variables;
- dependence or independence of random variables of request waiting time and service time;
- limited or unlimited value of the number of service stations, queue length;
- the system's service discipline.

The random variables which appear in the analysed model are of the type:

1. the time passed between the input requests;
2. the service time of one request by the service station;
3. the number of service stations;
4. the number of spots in the queue awaiting service.

A set of interconnected queue systems, where requests are transferred realizing the service demand, form a network of simple systems of mass service. It should be noticed that analogous elements to the transport network appear in the queue network, which makes it possible to apply concepts and claims from transport networks to describe the properties of queue networks (Table 1).

The most commonly used measures to assess the characteristics of a queue system in application to transport systems are:

- the side of flow determined on the basis of client service intensity within the system,
- indicators of congestion described as separation among clients or traffic density,

Table 1. Comparison of the transport network with queue network

Elements of the network	**Queue network**	**Transport network**
Input	input request (source)	vertex with only outgoing edges
Output	output request	vertex with only incoming edges
Flowrate	request stream	flow volume
Vertex	queue system	transport node
Edge	possible transfer	arc between nodes
Direction of the edge	direction of transfer	flow direction
Value describing the edge	probability of transfer $0<p\leq 1$	capacity

Source: developed based on [4]

– the number of lost clients measured by the number of clients who declined service due to a long wait time,
– the percent of clients waiting to be serviced measured by the percentage of clients which wait in the queue before they are serviced,
– cost of service determined by the length of the queue,
– system performance measured as a function of: queue length, system saturation, etc.

3. MODEL OF TRAINS MOVEMENT MAPPING ON THE RAILWAY NETWORK

The queuing theory can be used to analyse the train traffic on a selected part of the rail network for its liquidity - the train's ability to overcome the analysed routes without commercial stops (mainly for travellers exchange) For the purposes of the study, it is assumed that:

– travelling unit in the network is a train,
– trains begin their route on the initial station and terminate on the final station,
– elements of the model can be forwarding offices (stations and passenger stops), where trains can stop,
– analysis is carried out for a specified period of time - e.g. 24 hours, and adopts the following limitations:
– related to the track layout occurring at the stations and on the open lines (the number of tracks should be mapped at the operating control points and lines),
– travel time of trains between forwarding offices should be specified,
– stopping time in forwarding offices should be specified,
– traffic dependencies should also be appropriately mapped (control command and signalling on the stations and on the open lines, non-simultaneity of arrival and departure, the existence of track closures and closures of traffic stations, etc.).

Model of trains movement mapping can be created for different configurations of the stations and open lines. Fig. 1 shows an example of a section of a railway line which is analysed.

Fig. 1 shows a diagram of a section of a single track railway line. The model includes the beginning, final and two intermediate station and three single track open lines. Trains to pass the analysed segment can appear both at the beginning and final station according to the specified timetable. Individual elements of the line system may be reproduced using the simple queue systems.

For the single-way railway line there is a characteristic way of riding on the open line. At the moment there can be only one train on the open line. There are exceptions where self-locking devices are installed, where there may be more than one train on the open line but only in the same direction. There is always the possibility to ride trains from the opposite direction. Thus, the model of train traffic on a single track railway line, which is not equipped with an automatic line block system, can be presented as in Fig. 2.

For the analysed scheme the initial condition must be assumed: expedition of the train in one direction prevents expedition of the train from the opposite direction. In the real system it is done by telephone pre-announcement of trains or handling of automatic line block system. In the model shown in Fig. 2, the diagram traffic conduction from beginning station to station A is illustrated with the red arrows. The first queue station is the exit semaphore B from the beginning station. The train is waiting near it

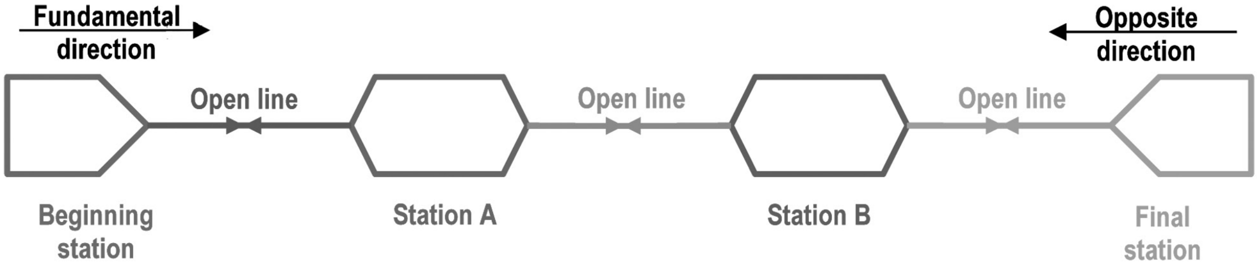

Fig. 1. Diagram of a section of a single track railway line

Source: own work.

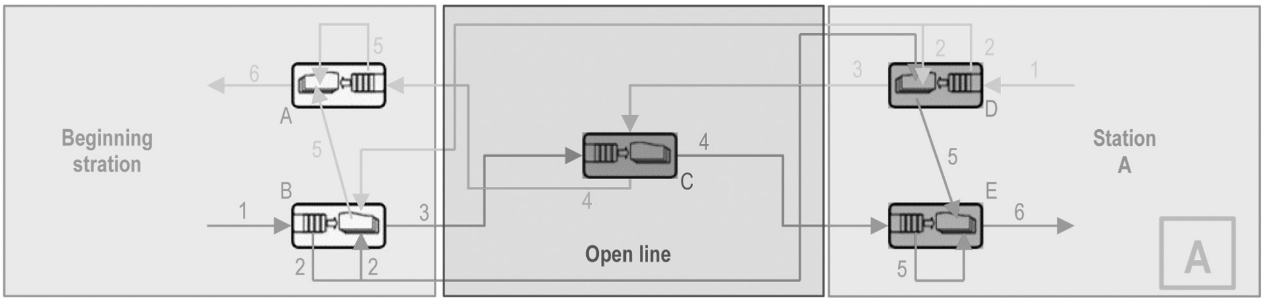

Fig. 2. Diagram of the traffic on the single track railway line without self-locking
Source: own work.

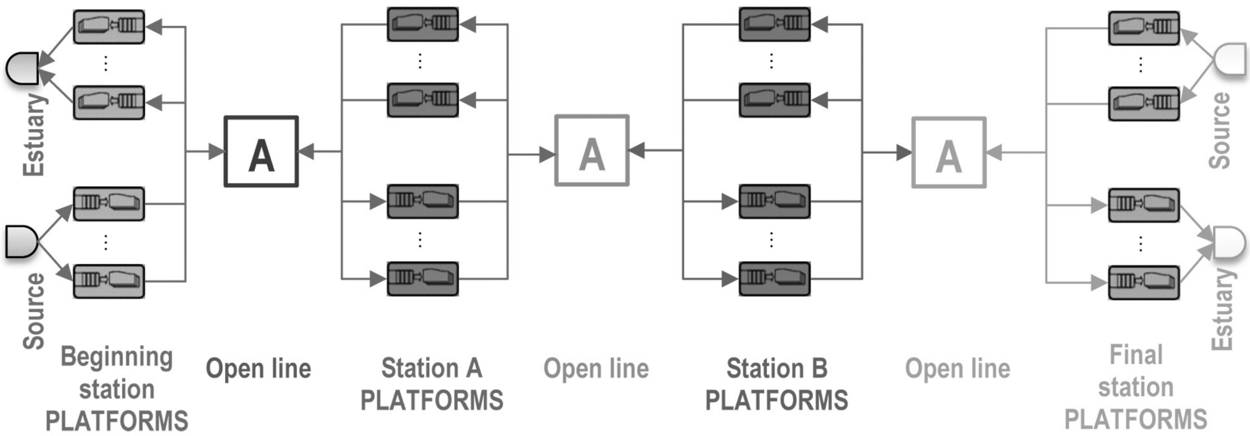

Fig. 3. Section of the railway line presented using simple queue systems
Source: own work.

for departure to the open line (1). Upon receiving the signal, the train will be operated in the queue system B (2) and prevention from leaving train from the opposite direction (hence the connection (2) from the system B will cause the occupancy of the service station in B and in D). The departure of the train begins (3), which terminates at the moment of covering the route by signal prohibiting driving on the semaphore. The train follows the route (it is operated) and after reaching the entrance semaphore of station A (4) begins its operation in the queue system E (5). Train operation, i.e. the display on the entry semaphore permit signal, is the same as the release of the operating position in the D system. This means that there is a possibility to pass the train in the opposite direction. After the service (covering the train by a forbidden signal), the train occupies the platform (6). The same sequence of actions can be distinguished for the opposite direction.

Fig. 3 shows the section of the railway line from Fig. 1 presented using simple queue systems. The set of elements shown in Fig. 2 is labelled by A in the square.

4. TESTING OF TRAFFIC FLUIDITY OF TRAINS ON THE SELECTED RAILWAY LINES USING THE QUEUE THEORY

As mentioned earlier, the subject of the article is the application of the queue theory to assess the implementation of transport processes on the example of railway traffic - the analysis was carried out on the process of trains moving due to traffic fluidity. The problem is searching for the minimal time for train calls from the Warsaw Grochow station and from the east lines, for which no queue is being constructed on the analysed railway network. In the article, the following open lines were conducted:

– Warsaw East – Warsaw West – railway line 448 (Warsaw West – Warsaw Rembertow) /tracks dedicated for suburban passenger traffic, direction opposite to the fundamental/,
– Warsaw East – Warsaw Central and Warsaw Central – Warsaw West – section of railway line 2 – Warsaw West – Terespol) / tracks dedicated for domestic passenger traffic, direction opposite to the fundamental /.

The diagram of the analysed sections of railway lines is shown in Fig. 4.

The analysed open lines are characterized by the following parameters:

1. Open line Warsaw East – Warsaw West (section of railway line 448):

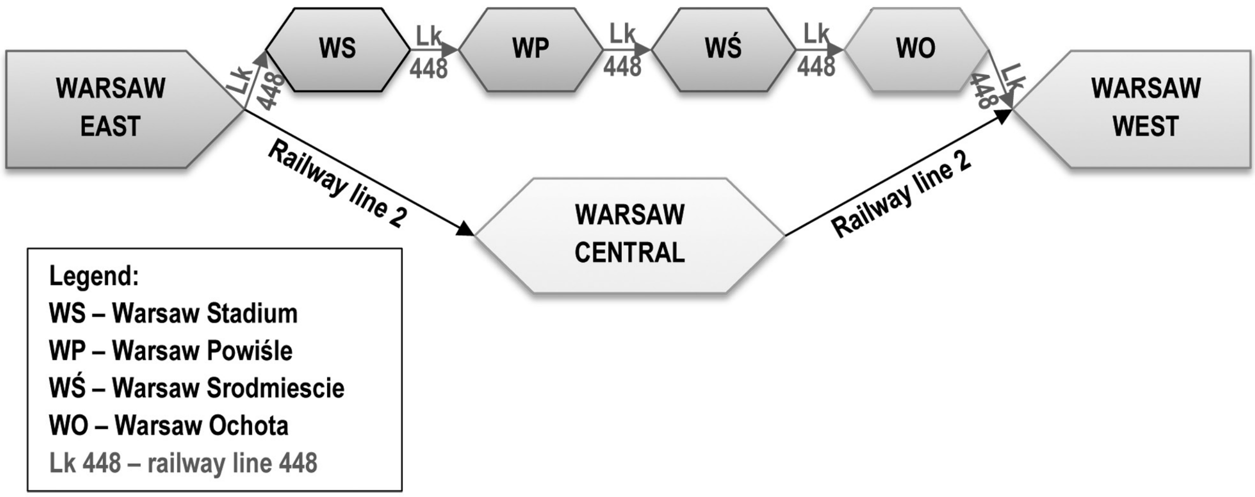

Fig. 4. Diagram of the analysed sections of railway lines
Source: own work.

- Length of the open line – 7,189 km;
- Average typical train travel time of the open line – 14 min (including stops on the passenger stops: Warsaw Stadium, Warsaw Powisle, Warsaw Srodmiescie and Warsaw Ochota);
- Type of linear motion control devices – four-aspect automatic line block system (split of the track at 11 spacing);
- Average length of the one spacing – 654 m;
- Average travel time of the one spacing – 1,3 min;
- Number of platform edges at beginning station – 3;
- Number of platform edges at final station – 3;

2. Open line Warsaw East – Warsaw West (section of railway line 2):

- Length of the open line – 4,254 km;
- Average typical train travel time of the open line – 6 min;
- Type of linear motion control devices – three-aspect automatic line block system (split of the track at 6 spacing);
- Average length of the one spacing – 709 m;
- Average travel time of the one spacing – 1 min;
- Number of platform edges at beginning station – 5;
- Number of platform edges at final station – 4.

For the purposes of the study, the model for the selected part of the real railway transport system was constructed for the above mentioned data. It is assumed that point elements of the railway infrastructure will be mapped by vertices, while elements of linear infrastructure by edges. The elements of the model mapping the elements of the railway network are presented in Table 2.

Fig. 5 shows graphical model of selected railway open lines in the form of simple mass-service network systems.

The parameters of the individual components of the model (see Fig. 5) in the basic state are presented in Table 3. The distribution of the request flow and service time was based on the analysis of the available train timetable (values were expressed in minutes).

Table 2. Elements of the model mapping the elements of the railway network

Type of element	Number of element v	Name of element
Source no 1		flow of trains reporting to the Warsaw East from the Warsaw Grochow station and from the east lines
Service stations	1	suburban platforms at the station Warsaw East
	2	1st interval of automatic line block system on the Warsaw East - Warsaw West open line (platforms of Warsaw Stadium passenger stop)
	3	2nd interval of automatic line block system on the Warsaw East - Warsaw West open line
	4	3rd interval of automatic line block system on the Warsaw East - Warsaw West open line
	5	4th interval of automatic line block system on the Warsaw East - Warsaw West open line (platforms of Warsaw Powisle passenger stop)
	6	5th interval of automatic line block system on the Warsaw East - Warsaw West open line
	7	6th interval of automatic line block system on the Warsaw East - Warsaw West open line
	8	7th interval of automatic line block system on the Warsaw East - Warsaw West open line (platforms of Warsaw Srodmiescie passenger stop)
	9	8th interval of automatic line block system on the Warsaw East - Warsaw West open line
	10	9th interval of automatic line block system on the Warsaw East - Warsaw West open line (platforms of Warsaw Ochota passenger stop)
	11	10th interval of automatic line block system on the Warsaw East - Warsaw West open line
	12	interval of automatic line block system before entering the group of suburban tracks on the station Warsaw West
	13	suburban platforms at the station Warsaw West
	14	long-distance platforms at the station Warsaw East
	15	1st interval of automatic line block system on the Warsaw East - Warsaw Central open line
	16	2nd interval of automatic line block system on the Warsaw East - Warsaw Central open line
	17	3rd interval of automatic line block system on the Warsaw East - Warsaw Central open line
	18	4th interval of automatic line block system on the Warsaw East - Warsaw Central open line
	19	5th interval of automatic line block system s on the Warsaw East - Warsaw Central open line
	20	interval of automatic line block system before entering to the station Warsaw Central
	21	platforms at the station Warsaw Central
	22	1st interval of automatic line block system on the Warsaw Central - Warsaw West open line
	23	2nd interval of automatic line block system on the Warsaw Central - Warsaw West open line
	24	interval of automatic line block system before entering to the station Warsaw West
	25	long-distance platforms at the station Warsaw West
Estuary no 1		flow of trains leaving Warsaw West to Warsaw Ochota and on the lines in the west side

Source: own work

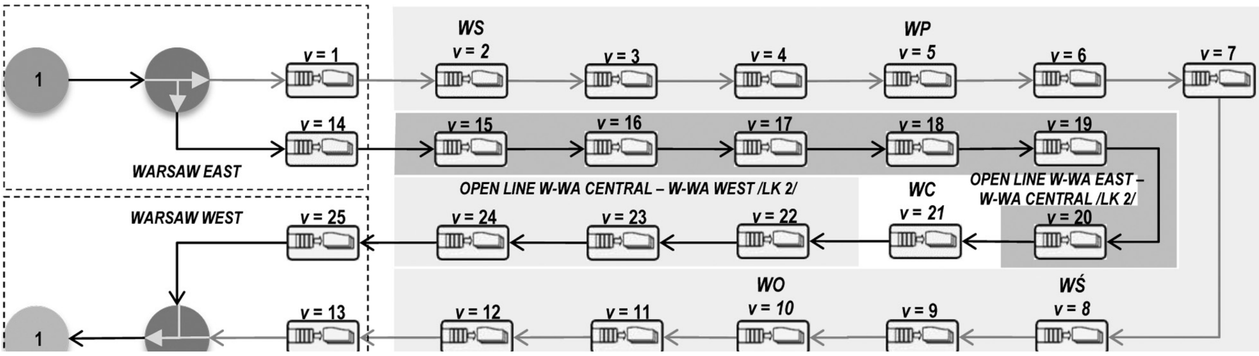

Fig. 5. Model of a railway line in the form of simple mass-service network systems (signs on the figure from table 2 and: W-WA – Warsaw, WS – Warsaw Stadium, WP – Warsaw Powisle, WC – Warsaw Central, WŚ – Warsaw Srodmiescie, WO – Warsaw Ochota)
Source: own work.

Table 3. Model parameters of the railway network developed using simple systems of mass service

Model element	Distribution of applications	Waiting capacity	Rules of the queue	Number of service stations	Distribution of service time
a = 1	erl(5,25)	–	–	–	–
v = 1	–	0	FIFO	3	norm(1,0.5)
v = 2	–	0	FIFO	1	norm(2,0.5)
v = 3, 4, 5, 6, 7	–	0	FIFO	1	norm(1,0.5)
v = 8	–	0	FIFO	1	norm(1.5,0.5)
v = 9, 10, 11, 12	–	0	FIFO	1	norm(1,0.5)
v = 13	–	0	FIFO	3	norm(1,0.5)
v = 14	–	0	FIFO	5	norm(3,1)
v = 15, 16, 17, 18, 19, 20	–	0	FIFO	1	norm(1,0.5)
v = 21		0	FIFO	4	norm(4,1)
v = 22, 23, 24		0	FIFO	1	norm(1,0.5)
v = 25		0	FIFO	4	norm(1,0.5)
b = 1	–	–	–	–	–

Source: own work

5. APPLICATION OF THE JAVA MODELING TOOLS COMPUTER PACKAGE - JSIM GRAPH FOR TRAFFIC FLUIDITY TESTS ON THE RAILWAY LINE

The Java Modelling Tools - JSIMgraph application was used to analyse the fluidity of train traffic on the analysed open lines of selected railway lines. Fig. 6 shows a model of the analysed routes on selected railway lines in the form of simple mass-service network systems using Java Modelling Tools.

The model entered into the program was parameterized on the basis of available data (e.g. train timetable). The values of parameters introduced into the model are presented in Table 3. Due to the specificity of the railway traffic, the distribution of train calls from the Warsaw Grochow station and the east lines was described by the Erlang distribution. To the distribution of the service time on individual stations we used the normal distribution.

The results of the performed simulations for different call distributions of trains are shown in Table 4.

On the basis of the obtained results, it can be stated that the number of serviced trains is not greater than unity at the time of train reporting of 5 minutes at any station. At stations (Warsaw East, Warsaw Central and Warsaw West) appeared queue times greater than zero. However, it is not greater than 0,05 minutes. This means that the fluidity of

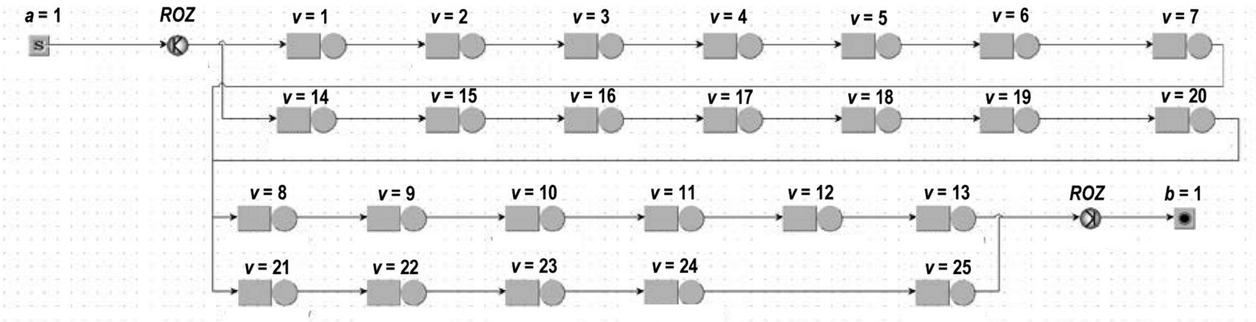

Fig. 6. Model of the analysed open lines on selected railway lines in the form of simple mass-service systems (signs in the figure from the table 2 and: ROZ - turnout)
Source: own work using Java Modelling Tools

Table 4. **T**he results of the performed simulations for different call distributions of trains

Model element	Call distributions of trains					
	erl(5.25)		erl(4.16)		erl(3.9)	
	Number of trains	Queue time	Number of trains	Queue time	Number of trains	Queue time
v = 1	0.206	0.0	0.258	**2.16E-4**	0.343	**6.82E-3**
v = 2	0.400	0.0	0.506	0.0	0.738	0.0
v = 3	0.206	0.0	0.259	0.0	0.345	0.0
v = 4	0.206	0.0	0.258	0.0	0.352	0.0
v = 5	0.207	0.0	0.260	0.0	0.356	0.0
v = 6	0.207	0.0	0.262	0.0	0.356	0.0
v = 7	0.207	0.0	0.262	0.0	0.360	0.0
v = 8	0.305	0.0	0.390	0.0	0.550	0.0
v = 9	0.207	0.0	0.261	0.0	0.352	0.0
v = 10	0.208	0.0	0.262	0.0	0.359	0.0
v = 11	0.207	0.0	0.263	0.0	0.362	0.0
v = 12	0.208	0.0	0.266	0.0	0.363	0.0
v = 13	0.209	**0.017**	0.264	**0.033**	0.365	**0.066**
v = 14	0.613	**0.054**	0.845	*0.369*	*82.818*	*12.003*
v = 15	0.205	0.0	0.258	0.0	0.347	0.0
v = 16	0.206	0.0	0.259	0.0	0.352	0.0
v = 17	0.207	0.0	0.261	0.0	0.353	0.0
v = 18	0.207	0.0	0.262	0.0	0.357	0.0
v = 19	0.208	0.0	0.261	0.0	0.360	0.0
v = 20	0.208	0.0	0.263	0.0	0.358	0.0
v = 21	0.208	**0.014**	0.263	**0.028**	0.360	**0.057**
v = 22	0.209	0.0	0.267	0.0	0.359	0.0
v = 23	0.209	0.0	0.265	0.0	0.363	0.0
v = 24	0.209	0.0	0.267	0.0	0.362	0.0
v = 25	0.209	**0.018**	0.265	**0.034**	0.365	**0.067**

Source: own work

traffic is preserved. Under the time of train reporting of the 4 minutes also the maximum number of simultaneously serviced trains at each station was not exceeded, while the time of the queue was increased and at one of the stations was equal to 0,37 minutes. It can be assumed that this time is acceptable and the flow of traffic is preserved. On the other hand, a queue of 82.8 trains and a queue time of 12 minutes took place at one reporting time of 2 minutes. This means that the movement is not smooth. In summary, the rational reporting time is a 3 minute stream.

6. SUMMARY

The mass service theory is often used to study processes in transport where queues are formed. One of the areas in which these types of problems arise is railway transport. The organization of railway traffic developed by the infrastructure manager assumes that all processes are completed according to a previously prepared plan. However, in real operation, many different disruptions may arise- for example, rail traffic control, command and signalling device malfunction. Using the queueing theory, it is possible to predict the results of these types of occurrences, and at the same time prepare loss-absorbing operations.

This article presented the application of the queueing theory to assess the realization of transport processes on the example of train movement based on traffic flow. The study was conducted on the basis of selected stretches of railway lines, numbers 2 (central long-distance in Warsaw) and 448 (central suburban in Warsaw). An ideal situation was considered, where all devices are functioning and there are no disruptions in traffic flow. The studies showed that the traffic on the railway line was continuous at the time. The average number of trains serviced at each respective station does not exceed 0,5, queues do not form and the average number of serviced trains at once in the system was near 4,85.

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